Compressive Spectral Anomaly Detection

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Abstract

We propose a novel compressive sensing imager for detecting anomalous spectral profiles in a scene. The sensing strategy models the background spectrum to lie on a low-dimensional subspace while assuming the anomalies to form a spatially-sparse set of spectral profiles different from the background. Our core contributions are in the form of a two stage sensing mechanism: in the first stage, we estimate the subspace for the spectrum of background by acquiring spectral measurements at a few randomly-selected pixels. In the second stage, we acquire spatially-multiplexed spectral measurements of the scene. Projecting the spectral measurements on the complementary subspace creates a sparse matrix which is then recovered using a Multiple Measurement Vector problem. Theoretical analysis and simulations show significant speed up in acquisition time over other anomaly detection techniques. A lab prototype based on a DMD and a visible spectrometer validates our proposed imager.

1. Introduction

Hyperspectral images measure light intensity in a scene as a function of space and spectrum. The availability of spectral measurements at each spatial location helps in identifying material composition. This has found applications in facial recognition [18, 14], segmentation [23], and numerous geoscience and remote sensing applications [9].

An important application of hyperspectral imaging is to identify the presence of anomalous materials in a scene. When the materials of interest are present in trace quantities, it is common to model them as anomalous spectral signatures [16]. The most common approach for hyperspectral anomaly detection is to represent the background using a low-dimensional subspace. Subsequently, a spatial location or a “pixel” which cannot be represented accurately by the background model is tagged as an anomaly. Estimation of the location of these anomalies typically requires acquisition of the complete hyperspectral image. Since typical hyperspectral images are extremely high-dimensional, the imaging effort is not only time consuming and also requires highly sensitive and expensive sensor elements.

In this paper, we propose a novel sensing framework that directly estimates the location of anomalous spectral signatures without sensing the entire hyperspectral image associated with a scene. We model the hyperspectral image as a sum of the background, assumed to be low-rank, and the anomalies, assumed to be sparse. Under these modeling assumptions, we estimate the locations of the anomalies using a computational camera that implements the following two-stage approach. In the first stage, we build a low-dimensional subspace for the background spectrum by acquiring a spectral measurements at a few randomly-selected spatial locations in the scene. In the second stage, we obtain spatially-multiplexed spectral measurements of the scene and estimate the sparse anomalies by projecting out the subspace corresponding to the background spectrum. Figure 1 shows estimation of 200 anomalous pixels of oil spill in $256 \times 256 \times 360$ Deepwater oil spill hyperspectral image [22] with 3500 spectral measurements. Our algorithm recovers anomalies with very high accuracy with fewer measurements than the dimension of the signal.
Our sensing framework enables a significant reduction in the number of measurements over competing schemes that require the entire hyperspectral image, thereby providing significant speedups and efficiencies at acquisition time.

We make the following contributions in this paper.

- **Novel sensing framework.** We propose a sensing framework that detects anomalies directly with fewer measurements than the dimension of the hyperspectral image.

- **Theoretical guarantees.** We provide expressions for the minimum number of measurements and recovery accuracy for our proposed sensing method.

- **Validation with a lab prototype.** We demonstrate the efficacy of our approach using a lab prototype that uses a digital micromirror device (DMD) for spatial modulation and a spectrometer for spectral measurements.

We also evaluate our technique, on synthetic data, against the classical Reed Xiaoli (RX) [21] detector as well as more recent techniques that rely on compressive imaging under the low rank and sparse signal model.

2. Prior work

**Notation.** We denote vectors in bold font, scalars in small letters and matrices in capital letters. The $\ell_0$-norm of a vector is the number of non-zero entries. Given a matrix $X \in \mathbb{R}^{M \times N}$ and an index set $\Omega \subset \{1, 2, \ldots, N\}$, $X_\Omega$ represents a matrix of size $\mathbb{R}^{M \times |\Omega|}$ formed by selecting columns of $X$ corresponding to the elements of $\Omega$. The $p,q$-norm of a matrix is defined as

$$
\|X\|_{p,q} = \left( \sum_{n=1}^{N} \|X_n\|^p \right)^{1/q}.
$$

Note that this is an entry-wise norm and is not to be confused with the induced matrix norms. Hence, $\|X\|_{2,2}$ is the Frobenius norm of a matrix, $\|X\|_{2,1}$ is the sum of $\ell_2$-norms of columns, $\|X\|_{2,0}$ is the number of non-zero columns of $X$ and $\|X^T\|_{p,0}$ is the number of non-zero rows of $X$. Consider a signal $x \in \mathbb{R}^{JN}$, which can be reshaped into a $J \times N$ matrix, $X$. The vector $x$ is called $K$-block sparse if the number of non-zero columns in $X$ is no more than $K$.

**Compressive sensing (CS).** CS aims at recovering a signal from a set of linear measurements fewer than its dimensionality [3], i.e., we seek to acquire a signal $x \in \mathbb{R}^N$ from $M \ll N$ measurements, $y = \Phi x + e$, where $\Phi \in \mathbb{R}^{M \times N}$ is the sensing matrix, and $e$ is the measurement noise. CS relies on the sensed signal being sparse either canonically or under a linear transformation. In such a scenario, we can recover the signal by solving

$$
\min_x \|y - \Phi x\|_2, \quad \|x\|_0 \leq K.
$$

There are many approaches to solving this optimization problem, including convex relaxation techniques like Basis Pursuit [24] and greedy techniques like Orthogonal Matching Pursuit [19] and CoSaMP [17]. This model is also called the single measurement vector (SMV) problem.

The multiple measurement vector (MMV) problem extends the idea of sensing of sparse signals to a scenario where we sense multiple signals that share a common sparsity pattern [10]. The MMV problem can be formulated as

$$
Y = \Phi X + E, \quad \|X^T\|_{2,0} \leq K,
$$

with $Y \in \mathbb{R}^{M \times Q}, X \in \mathbb{R}^{N \times Q}, \Phi \in \mathbb{R}^{M \times N}$ and $E \in \mathbb{R}^{M \times Q}$. Here, $Y = [y_1, y_2, \ldots, y_Q]$ are the multiple measurements, $X = [x_1, x_2, \ldots, x_Q]$ are the signals that share the same sparsity pattern, and $E = [e_1, e_2, \ldots, e_Q]$ is the measurement noise. Solutions to the MMV problem can be obtained through algorithms similar in spirit to the SMV problem [10]. The MMV model and the results in Baraniuk et al. [4] are of particular interest to the results in this paper since CS of spatially sparse hyperspectral image under spatial multiplexing reduces to an MMV problem, with each spectral image representing a signal of interest.

**Hyperspectral image models.** Hyperspectral images can be seen as a composition of a background composed of a few spectral profiles and anomalies, with spectral profiles different from that of the background. Hence, the background spectrum can often be represented by a low-dimensional model and anomalies can be represented as spatially sparse outliers. A sum of low-rank and sparse matrix models is therefore very appealing for hyperspectral images and is of particular interest to us, as it is simple and represents the background spectrum and anomalies accurately. Consider a hyperspectral image with spatial resolution of $N_1 \times N_2$ pixels and a spectrum resolution of $N_3$ spectral bands. We represent such a hyperspectral image as a $N_3 \times N_1 N_2$ matrix such that each column of this matrix is the spectral profile at a particular pixel and each row representing the image at a particular spectral band. Under this notation, the low rank and sparse matrix model is given as

$$
X = L + S, \quad \text{rank}(L) \leq k, \quad \|S\|_{2,0} \leq T \quad (1)
$$

Chakrabarti et al. [7] showed that hyperspectral images of natural scenes are very well represented by this signal model. The low rank and sparse model has been used for various hyperspectral imaging tasks [27, 12, 29]. While such models have found use in CS of hyperspectral images with anomaly detection in post-processing, there are few results in existing literature that directly sense the spectral anomalies directly from compressive measurements.

**Hyperspectral imaging.** Hyperspectral images are captured by various techniques like spatial scanning, spectral
scanning, snapshot imaging and spatio-spectral scanning [25]. Since we need to measure the spectral profile at each pixel, the acquisition process is extremely time consuming and wasteful if we are only interested in detecting anomalies. Hence, CS approaches have been pursued to reduce the number of acquired measurements [1, 26].

Identifying anomalous spectral signatures in hyperspectral images is done in the post-process stage, which involves techniques like hyperspectral unmixing [5], support vector data descriptor [2] and subspace-based methods [20]. The idea underlying these methods is to fit a model to the background spectrum in the hyperspectral image; any pixel which does not conform to this model is classified as an anomalous spectral signature. These techniques require either measuring the whole hyperspectral cube, or recovery of the whole cube from its compressive measurements.

We propose a sensing technique that directly estimates the location of spatially sparse anomalies from a few compressive measurements. Such a model, which involves sensing of spatially sparse signals by removing contribution for background has been explore in video surveillance [6]. Our paper extends this idea to hyperspectral images by knocking off the contribution from background spectra, which results in a spatially sparse hyperspectral image. Locations of the anomalous spectral signatures can then be estimated by solving it as an MMV problem.

3. Proposed methodology

We now detail the imaging architecture for obtaining measurements for the proposed method and provide details of algorithm to solve for the anomalous pixel locations.

3.1. Sensing architecture

Our optical setup (see Figure 2) consists of a DMD and a spectrometer. The scene is focused onto the DMD using an objective lens. Subsequently, light directly from the DMD along one of the mirrored optical axis is fed onto a spectrometer. If we represent the hyperspectral image $X$ as a matrix of dimensions $N_3 \times N_1 N_2$ such that each column of the matrix is the spectral profile at a pixel, then a measurement $y_i \in \mathbb{R}^{N_3}$ at the spectrometer is of the form $y_i = X \phi_i + e_i$, where $\phi_i \in \mathbb{R}^{N_1 N_2}$ is the binary pattern displayed on the DMD and $e_i$ is the measurement noise. It is worth noting that this optical system is the same as a single pixel camera [11], except that the photodetector (i.e., the single pixel) has been replaced by a spectrometer. Hyperspectral imaging using such an architecture has been explored in [15].

We can obtain multiple set of spectral measurements by displaying different patterns on the DMD. The resulting measurement model can be expressed as $Y = X \Phi + E$, where $Y = [y_1, y_2, \ldots, y_M]$ is the collection of spectral measurements, $\Phi \in \mathbb{R}^{N_1 N_2 \times M}$ is the sensing matrix and $E \in \mathbb{R}^{N_3 \times M}$ is the measurement noise.

3.2. Anomaly detection algorithm

We model a hyperspectral image $X$ as a sum of a low rank matrix $L$ and a sparse matrix $S$. Our aim is to estimate the non-zero columns of $S$ with as few measurements as possible, since each column represents the spectrum at a particular scene pixel. We achieve this using a two-stage sensing mechanism. In the first stage, we obtain a low-rank subspace $U$ corresponding to the column-span of the matrix $L$; this subspace provides a model for the background spectra. In the second stage, we obtain a few spatially-multiplexed measurements. Complementary projection onto the previously estimated subspace leaves only the sparse outliers, which are recovered using sparse approximation techniques.

Stage 1 – Subspace estimation. The goal is to estimate a subspace for the background pixels by acquiring $n_1$ spectrometer measurements. Let $\Omega_1$ represent the indices of $n_1$ randomly-chosen spatial locations. We acquire measurements of the form $Y_1 = X_{\Omega_1} + E_1$, where $E_1 \in \mathbb{R}^{N_3 \times n_1}$ is the measurement noise. We estimate a basis $U \in \mathbb{R}^{N_3 \times k}$ for the $k$-dimensional subspace, such that

$$Y_1 = L_{\Omega_1} + S_{\Omega_1} + E_1 = U \Lambda_{\Omega_1} + S_{\Omega_1} + E_1, \quad (2)$$

We estimate the basis $U$ by solving (2) under a Robust PCA [28] formulation.

Stage 2 – Finding outliers. In the second stage, we obtain $M$ spatially-multiplexed measurements of the form

$$Y_2 = X \Phi + E_2 = L \Phi + S \Phi + E_2, \quad (3)$$

![Figure 2: Our built imager consists of a DMD which displays desired patterns and a spectrometer which obtains spectrum of the scene’s dot product with DMD pattern.](image-url)
Since we have estimated the basis for the low rank representation in the first stage, we can write $L = UΛ$, $Λ ∈ \mathbb{R}^{K×N_1N_2}$. Thus $Y_2 = UΛΦ + SP + E_2$. Let $P_U^\perp$ be the complementary projection operator for $U$.

$$Y = P_U^\perp Y_2 = P_U^\perp UΛ + P_U^\perp SΦ + P_U^\perp E_2 = 0 + P_U^\perp SΦ + P_U^\perp E_2 = \tilde{S}Φ + \tilde{E}_2. \quad (4)$$

Note that $\tilde{S}$, which is the projection of the sparse matrix onto the complementary subspace of $U$, retains its sparsity structure, as the projection operation is only for the spectra, spanned by the columns. Estimating $\tilde{S}$ given $Y$ and $Φ$ is then an MMV problem [10].

### 4. Recovery guarantees

We provide guarantees on recovery in terms of minimum number of measurements and estimation error.

#### 4.1. Minimum number of measurements

We now provide an estimate of the minimum number of measurements required to estimate the anomalies with our sensing strategy. We rely heavily on results from model-based compressive sensing [4] and Chen et al. [8].

**Proposed method.** To estimate the column span of the low-rank component, we need measurements proportional to its rank and hence, we need $O(kN_3)$ samples for stage-one. Stage-two requires compressive measurements of a block sparse signal. Baraniuk et al. [4] have shown that for a block sparse signal of size $J × N$, with $K$ non-zero blocks, measurements as few as $O(JK + K \log(N/K))$ are sufficient for robust recovery, as long as the measurement matrix satisfies sub-Gaussianity. In our setup, we have $J = N_3$ and $K = T$. Hence, the overall measurement rate of the proposed technique is

$$M_{\text{proposed}} \gtrsim (kN_3 + N_3T + T \log(N_1N_2/T)), \quad (5)$$

where $\gtrsim$ implies greater-than upto a multiplicative constant.

**Compressive sensing.** Chen et al. [8] showed that for a matrix of size $m × n$ which can be expressed as a sum of a rank-$k$ matrix and sparse non-zero columns, robust recovery is guaranteed with high probability under the following conditions,

$$p \gtrsim \frac{r \log^2(m + n)}{\min(m,n)} \quad \text{and} \quad \gamma \lesssim \frac{p}{k\sqrt{T} \log^3(m + n)}.$$  

$p$ is the fraction of observations and $\gamma$ is the fraction of non-zero columns in the sparse matrix. Adapting it to our method, we have

$$M_{\text{joint}} \gtrsim \frac{N_1N_2N_3r \log^2(N_1N_2 + N_3)}{\min(N_1N_2, N_3)} = rN_1N_2 \log^2(N_1N_2 + N_3),$$

where $M_{\text{joint}}$ is the number of measurements for joint recovery of sparse and low rank matrices. It is clear that the number of measurements for our proposed method is less than the number for joint recover of sparse and low-rank matrices. As an example, consider a hyperspectral image of size $256 × 256 × 300$ and 100 outliers. Let us assume the background is approximated by a rank-5 matrix. The number of measurements needed for the proposed sensing technique would be approximately $10 × 5 × 300 + 100 × 300 + 100 × 16 = 33,100$, whereas that for compressive sensing would be approximately $5 × 256 × 256 × 16 = 524,288$. This provides us a gain of $15$ in number of measurements.

#### 4.2. Recovery guarantees for outliers

We look at three conditions for recovery of outliers: noiseless measurements with knowledge of the exact subspace estimate, noisy measurements with knowledge of the exact subspace estimate, and noisy measurements with an inexact, rank-$k$ approximation of the subspace. The bounds and guarantees are based on the MMV variant of CoSaMP and we will repeatedly invoke the following result from [4].

**Theorem 1.** (Adapted from Theorem 6 in [4]) Let $S$ be a $K$ block-sparse signal and let $Y = SΦ + E$ be a set of noisy compressive sensing measurements, where $Φ ∈ \mathbb{R}^{N_1N_2×M}$. If $Φ$ is a random sub-Gaussian matrix, then the estimate obtained from iteration $t$ of block-based CoSaMP satisfies

$$\|S - \tilde{S}_t\|_{2,2} \leq \frac{1}{2t} \|S\|_{2,2} + 20 \left(\|S - S_T\|_{2,2} + \cdots \right) + \frac{1}{\sqrt{T}} \|S - S_T\|_{2,1} + \|E\|_{2,2}, \quad (6)$$

where $S_T$ is the best $T$-term approximation of $S$, and $E$ is the additive measurement noise.

In essence, the theorem states that recovery is guaranteed if $M$ is sufficiently large. Further details about the sensing matrices and number of measurements can be obtained in [4]. Next, we show the bounds on error with the three conditions of measurements.

**Noiseless and exact subspace.** In this case, we have the following measurement model, $\tilde{Y} = P_U^\perp Y$, where the sparsity of $S$ is the same as $S$. Invoking (6), we have

$$\|S - \tilde{S}\|_{2,2} \leq \frac{1}{2t} \|S\|_{2,2}, \quad (7)$$
where $\tilde{S} = [\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_{N_1N_2}]$ is the projection of $S$ onto the complementary subspace of $U$ and $\tilde{S}_i$ is the estimate of $\tilde{S}$ after $i$ iterations of MMV CoSaMP.

**Noisy and exact subspace.** Assume that the added noise is bounded as, $\|E_2\|_{2,2} \leq \epsilon \|\tilde{S}\|_{2,2}$. Here, $\epsilon$ is a measure of the signal-to-noise ratio. As before, taking complementary projection, we have $\tilde{Y} = \Phi + \tilde{E}_2$. Invoking (6), we have

$$
\|\tilde{S} - \tilde{S}_i\|_{2,2} \leq \frac{1}{2\epsilon} \|\tilde{S}\|_{2,2} + 20\|\tilde{E}_2\|_{2,2} \\
\leq \left(\frac{1}{2\epsilon} + 20\epsilon\right) \|\tilde{S}\|_{2,2}.
$$

**Noisy and inexact subspace.** Let $\hat{U}$ be the estimated rank $k$ estimate of the columns of $U$ and $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{N_1N_2}]$ represent coefficients for each column. Further, let the following bound hold

$$
\|P_U^T U \lambda_k\| \leq \epsilon_s \|\tilde{s}_k\|,
$$

where $\epsilon_s$ is a small constant. Also, let $\|\Phi\|_{2,2} \leq \eta$. Now taking the complementary projection as before, we have

$$
P_U^T \tilde{Y} = P_U^T U \Phi + P_U^T S \Phi + P_U^T E_2 \\
= E_s + \tilde{S}_2 \Phi + \tilde{E}_2,
$$

where $E_s = P_U^T U \lambda \Phi$. $P_U^T U \lambda$ can be treated as the error in $T$-term approximation of $\tilde{S}$. The $\|\cdot\|_{2,2}$ and $\|\cdot\|_{2,1}$ error bounds on this term,

$$
\|E_s\|_{2,2} = \left(\sum_{k=1}^{[\Omega_2]} \|P_U^T U \lambda_k\|_2^2\right)^{1/2}
$$

$$
\leq \left(\sum_{k=1}^{[\Omega_2]} \|P_U^T U \lambda_k\|_2^2\right)^{1/2}
$$

$$
\leq \left(\eta^2 \sum_{k=1}^{[\Omega_2]} \|\tilde{s}_k\|_2^2\right)^{1/2} = \eta \epsilon_s \|\tilde{S}\|_{2,2}.
$$

Similarly, we have

$$
\|E_s\|_{2,1} \leq \epsilon_s \|\tilde{S}\|_{2,1}.
$$

Invoking (6), we get

$$
\|\tilde{S} - \tilde{S}_i\|_{2,2} \leq \frac{1}{2\epsilon} \|\tilde{S}\|_{2,2} + \frac{20}{\sqrt{T}} \|E_s\|_{2,1} + \|\tilde{E}_2\|_{2,2} \\
\leq \left(\frac{1}{2\epsilon} + 20\epsilon + \epsilon_s\right) \|\tilde{S}\|_{2,2} + 20\epsilon_s \frac{1}{\sqrt{T}} \|\tilde{S}\|_{2,1},
$$

Equation (14) establishes the effect of model mismatch as well as measurement noise on our estimation algorithm. Provided that the model is well estimated and the noise energy is bounded, the estimated anomalous locations can be very accurate. The synthetic and real experiments section gives a brief insight into the effect of noise and model error.

**_joint recovery.** Ha et al. [13] discussed error bounds for recovery of low rank and sparse matrix from compressed and noisy data. For brevity, we skip the complete expression of error and request readers to refer to [13] for analysis on noise. Table 1 summarizes the number of measurements and the measurement noise for various methods.

**5. Synthetic experiments**

We compare our proposed technique against two compressive sensing techniques for recovering data from a low rank and sparse matrix, SPaRCS [27], which is a greedy technique and ADMM implementation of Robust PCA [28]. We create synthetic data by decomposing the data as a sparse and low rank component using Robust PCA and then adding the two together. Figure 3 shows the two hyperspectral images with anomalous locations highlighted as red pixels.

**Recovery accuracy.** We performed recovery of the anomalous locations with $n_1 = 500$ measurements to obtain subspace and by sweeping $n_2$ from 1,000 to 15,000 random-permuted Hadamard measurements. For comparison, we recovered a rank-6 matrix as well as a columnsparse matrix from the compressive measurements using SPaRCS and Robust PCA under corrupted columns model [8]. We define success ratio as the fraction of anomalous locations correctly identified. Figure 4 and 5 compares the success fraction for both the hyperspectral images under two different noise conditions. Our method outper-
Table 1: Comparing various methods based on number of measurements and the estimation error in anomalous pixels. Our proposed method requires far fewer measurements than even compressive sensing.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of measurements</th>
<th>Bounds on recovery accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full measurements</td>
<td>$N_1 N_2 N_3$</td>
<td>Measurement noise</td>
</tr>
<tr>
<td>Compressive sensing</td>
<td>$O(r N_1 N_2 \log^2 (N_1 N_2 + N_3))$</td>
<td>Joint low rank and sparse recovery [13]</td>
</tr>
<tr>
<td>Proposed method</td>
<td>$O(r N_3 + N_3 T + T \log(N_1 N_2 / T))$</td>
<td>$a |S|_2 + b T |S|_1$</td>
</tr>
</tbody>
</table>

Effect of noise and number of measurements. Table 2 shows success fraction as a function of noise and the number of initial measurements for a $128 \times 128 \times 366$ dimensional hyperspectral image with 71 anomalous pixels. It is evident that noise plays a significant role in estimation of outliers. For estimating the subspace, measurements greater than 400 are sufficient for good accuracy. Effect of high noise can also be seen in Figure 5b, with Robust PCA and SPArC outperforming our method for large number of measurements. This is consistent with our theoretical guarantees, where we showed in (14) that there is a significant effect of measurement noise on estimation accuracy.
### Table 2: Percentage of correctly detected targets as a function of SNR and number of initial measurements for 128 × 128 × 356 hyperspectral image of an urban scene.

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>( n_1 = 100 )</th>
<th>( n_1 = 400 )</th>
<th>( n_1 = 800 )</th>
<th>( n_1 = 1000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>60.06</td>
<td>68.99</td>
<td>68.20</td>
<td>70.79</td>
</tr>
<tr>
<td>60</td>
<td>88.59</td>
<td>91.92</td>
<td>92.37</td>
<td>91.66</td>
</tr>
<tr>
<td>80</td>
<td>96.59</td>
<td>98.53</td>
<td>98.28</td>
<td>98.31</td>
</tr>
</tbody>
</table>

![Image of a hyperspectral camera setup](image)

**Figure 7:** Setup for DMD-based hyperspectral imaging. The objective lens (A) focuses light onto the DMD (B). The light is then measured by the spectrometer consisting of a condenser lens (E) that focuses light onto an iris (F), which is then collimated by the lens (G). The collimated light is diffracted by a reflective diffraction grating (H), which is then focused by lens (I) onto the line sensor (J).

### 6. Real data experiments

To validate our proposed sensing method, we built a visible (400-700nm) hyperspectral camera based on architecture proposed by Li et al. [15] (see Figure 7). We show three examples of anomaly detection to demonstrate success of our proposed method.

#### 6.1. Experiment on data from SpecTIR

We performed anomaly detection on Deepwater Horizon oil-spill data obtained from SpecTIR [22], by obtaining 500 random pure spectral measurements for estimating subspace for background spectra and then obtaining 3000 spatially multiplexed measurements for estimating a total of 200 outliers. Figure 1 shows the 256 × 256 × 360 hyperspectral image we used for estimating outliers and the result of our algorithm. We observe that except a few points, all the estimated anomalies lie on the oil slick locations, identified by gray colored streaks in the image.

#### 6.2. Experiment on data from our imaging setup

We tested our algorithm on two scenes to estimate 5-8 outliers with the optical setup we built. We performed anomaly detection on 64 × 64 dimensional images with 50 spectral bands between 350 – 820nm.

**Subspace estimation.** Instead of obtaining subspace by randomly sampling the scene, we initially acquire a very low spatial resolution hyperspectral image, as this maximizes light throughput. If only a few anomalies are present, this method would still give a very good estimate of the subspace. In our experiments, we estimated subspace by taking an 8 × 8 hyperspectral image of the scene.

**Scene 1.** In our first experiment, we tested our sensing method on a scene made of “ICCP 2017” printed on a white paper with a hole in the letter “P”. A white LED is situated behind a diffuser at the location of this hole. Detected anomalies are shown in red in image (b).

**Scene 2.** In our second experiment, we tested our sensing method on a scene printed on paper with primarily red colors and four pixels illuminated by a laser, as show in...
Figure 9: Spectral signatures at some representative locations for the first scene. The noticeable peak of the anomalous pixel is due to a white LED.

Figure 10: Output of our algorithm on second real scene (b). The scene is made of a printed paper with primarily red color and four of the pixel locations are shone by a red laser. It is not easy to discern the location of laser with naked eye, but our algorithm identifies the anomalous locations with only 100 spectral measurements, shown as green dots in image (b).

Discussion. Success of anomaly detection with subspace modeling for the background relies on obtaining a good estimate of the subspace. A lower rank representation would not sufficiently represent the background which would give erroneous results. Since we estimated subspace from a downsampled version of the hyperspectral image, the spectrum of anomalous pixels may show up if higher rank is used. Figure 12 shows two failure cases with a lower and a higher rank subspace. Figure 13 shows residue after removing contribution from background for a subspace of rank 5, 8 and 12. Under estimation of rank gives error in regions other than the anomalies and an over estimate removes contribution due to anomalies. This problem exists with all anomaly detection algorithms. Erroneous representation of background spectra results in error in anomalies detected.

Prior knowledge of the complexity of the scene is definitely of help to estimate the rank of subspace. Further, smoother profiles are harder to differentiate. In our two examples, the stark difference between spectrum of laser and background made it easier to estimate outliers with a 4 dimensional subspace. On the other hand, the spectrum of white LED is smooth, making it harder to differentiate.

7. Conclusion

We presented a novel sensing method that directly obtains anomalies in a hyperspectral image without requiring the measurement of the complete image. Simulations and theoretical guarantees are strongly in favor of our proposed method. We also demonstrated a lab prototype that successfully detected anomalies in the scene. The method is particularly appealing as it detects anomalies with few measurements. As a consequence, we can develop imagers for geological surveillance that have higher temporal resolutions or better noise properties enabled by temporal averaging.

8. Acknowledgements

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